A M system modelling membrane formation, cytoskeleton growth and reproduction

The M system described in this section is inspired by the process of mitosis and cell replication, controlled by the growth of cytoskeleton, as illustrated schematically at Fig. 2. Observe that, even if this process relies on several relatively complex regulation mechanisms, the M system model can simulate it, at least at the morphological and dynamical level, with as few as 16 rules. All processes in the model are controlled by fully local interactions between its elements.

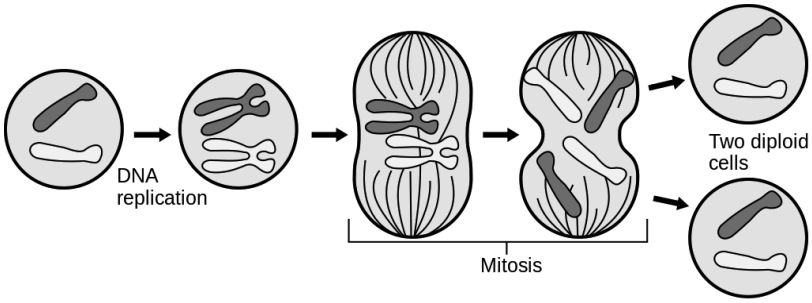

Fig. 2. Major events in mitosis. Licensed under Wikimedia Commons, authored by Mysid.

Let us start the formal description of the model by its polytopic tile system $T_0 = (Q, G, \gamma, d_g, S)$ in $\mathbb{R}^3$, which determines its spatial structure. Let

Q $= \{q_0, q_1, q_2, q_3, q_4, s_0, s_1, s_2\}$, where q_0, q_1, q_2 are larger pentagonal tiles forming cellular membrane, and q_3, q_4 are small pentagonal tiles forming nuclear membrane. All tiles contain facet connectors at all edges for mutual interconnection. There are three rods: s_0 and s_1 with two connectors at their endpoints, and s_2 is a rod with one connector at one endpoint and two fork-oriented connectors at the other endpoint. Rod s_0 is used only in the first step of the system. Rods s_1 and s_2 form straight and fork segments of cytoskeleton, respectively, and we call them *microtubules*. Furthermore,

- tile q_0 contains at the centre a point connector to which rod s_0 can connect by its endpoint connector;
- tile tile q_3 contains at the centre a point connector which can connect to the other end of rod s_0;
- tiles q_0 and q_1 contain at the centre a point connector to which rod s_1 can connect.

Let

Δ_p be a regular pentagon with distance 10 from center to vertices, with vertices denoted by $\mathbf{x}_1, \ldots, \mathbf{x}_5$;

Δ'_p be a regular pentagon with distance 3 from center to vertices, with vertices denoted by $\mathbf{x}'_1, \ldots, \mathbf{x}'_5$);
$\Delta_0 = \langle 0, 9.184 \rangle$ be a segment with length 9.184;
$\Delta_s = \langle 0, 1.4 \rangle$ be a segment with length 1.4.

The tiles in Q are defined as follows:

$$
\begin{aligned}
q_0 = (&\Delta_p, && \text{(pentagon)}\\
&\{(\langle \mathbf{x}_i, \mathbf{x}_{i \bmod 5+1} \rangle, g_e, \varphi_p) \mid 1 \le i \le 5\} && \text{(facet connectors)}\\
&\cup \{(\mathbf{0}, g_0, \pi/2), (\mathbf{0}, g_3, \pi/2)\}, && \text{(point connectors)}\\
&g_x), && \text{(surface glue)}
\end{aligned}
$$

$$
\begin{aligned}
q_1 = (&\Delta_p,\\
&\{(\langle \mathbf{x}_i, \mathbf{x}_{i \bmod 5+1} \rangle, g_a, \varphi_p) \mid 1 \le i \le 5\}\\
&\cup \{(\mathbf{0}, g_3, \pi/2)\}, g_x),
\end{aligned}
$$

$$
\begin{aligned}
q_2 = (&\Delta_p,\\
&\{(\langle \mathbf{x}_1, \mathbf{x}_2 \rangle, g_f, \varphi_p), (\langle \mathbf{x}_2, \mathbf{x}_3 \rangle, g_b, \varphi_p), (\langle \mathbf{x}_3, \mathbf{x}_4 \rangle, g_c, \varphi_p), (\langle \mathbf{x}_4, \mathbf{x}_5 \rangle, g_d, \varphi_p),\\
&(\langle \mathbf{x}_5, \mathbf{x}_1 \rangle, g_b, \varphi_p)\}, g_x),
\end{aligned}
$$

$$
\begin{aligned}
q_3 = (&\Delta'_p, \{(\langle \mathbf{x}'_i, \mathbf{x}'_{i \bmod 5+1} \rangle, g_a, \varphi_p) \mid 1 \le i \le 5\}\\
&\cup \{(\mathbf{0}, g_2, -\pi/2),\\
&g_t),
\end{aligned}
$$

$$
\begin{aligned}
q_4 = (&\Delta'_p,\\
&\{(\langle \mathbf{x}'_1, \mathbf{x}'_2 \rangle, g_f, \varphi_p), (\langle \mathbf{x}'_2, \mathbf{x}'_3 \rangle, g_b, \varphi_p), (\langle \mathbf{x}'_3, \mathbf{x}'_4 \rangle, g_c, \varphi_p), (\langle \mathbf{x}'_4, \mathbf{x}'_5 \rangle, g_d, \varphi_p),\\
&(\langle \mathbf{x}'_5, \mathbf{x}'_1 \rangle, g_b, \varphi_p)\}, g_x),
\end{aligned}
$$

$$s_0 = (\Delta_0, \{(\{0\}, g_1, \pi/2), (\{9.184\}, g_1, \pi/2)\}, g_x);$$

$$s_1 = (\Delta_s, \{(\{0\}, g_3, 0), (\{1.4\}, g_4, 0)\}, g_x);$$

$$s_2 = (\Delta_s, \{(\{0\}, g_5, 0), (\{1.4\}, g_6, \pi/30), (\{2\}, g_4, -\pi/30)\}, g_x);$$

where $\varphi_p = 2.034443935795703$ rad is the inner angle between two faces of a dodecahedron.

$G = \{g_0, g_1, g_2, g_3, g_4, g_5, g_6, g_a, g_b, g_c, g_d, g_e, g_f, g_t, g_x\}$ is the set of glues;
$\gamma = \{(g_0, g_1), (g_1, g_2), (g_3, g_3), (g_4, g_5), (g_6, g_3), (g_4, g_t), (g_6, g_t), (g_a, g_f), (g_f, g_a), (g_b, g_b), (g_c, g_c), (g_d, g_d), (g_e, g_f)\}$ is the glue relation;
$d_g = 0.1$ is the glue distance;
$S = \{q_0\}$ is the seed tile.

Then let us define the M system $\mathcal{M}_0 = (F, P, T_0, \mu, R, r, \sigma)$ such that:

$F = (O, m, \epsilon)$, where:
$O = \{a, b, c, x\}$ are floating objects;
$m(a) = 5$, $m(b) = 5$, $m(c) = 4$, $m(x) = 7$ is the mobility of floating objects;
$\epsilon(a) = 0.1$ and $\epsilon(o) = 0$ for all other $o \in O$ is the concentration of floating objects in the environment;
$P = \{p_0, p_1, p_2, p_3, p_4\}$;
T_0 is the polytopic tile system described above;

$\mu(q_2) = \{(p_0,(2,0)),(p_0,(2,2)),(p_2,(0,0)),(p_2,(0,2)),\} \cup \{(p_1,(i,j)) \mid i,j \in \{-5,5\}\}$, (proteins placed on tiles)
$\mu(q_4) = \{(p_3,(i,j)) \mid i,j \in \{-1,1\}\} \cup \{(p_4,(i,j)) \mid i,j \in \{0,1\}\}$,
$\mu(t) = \emptyset$ for all other $t \in Q$;
R contains the following rules:

Metabolic rules:
$a[p_0 \to [p_0 a$;
$[p_1 c \to [p_1 a$;
$[p_1 cx \to [p_1 aa$;
$[p_2 a \to [p_2 b$;
$[p_3 cx \to [p_3 aa$;
$cx[p_3 \to aa[p_3$;
$[p_4 a \to [p_4 cc$;

Creation rules:
$a^8 \to q_1$; (cellular membrane tiles creation from eight objects a)
$a^8 \to q_2$;
$aaa \to q_3$; (nuclear membrane tiles creation from three objects a)
$aaa \to q_4$;
$aaa \to s_0$; (auxiliary rod creation)
$b \to s_1$; (microtubules creation)
$b \to s_2$;

Division rules:
$g_c \overset{x}{-} g_c \to g_c, g_c$ (division of both cellular and nuclear membrane)
$g_d \overset{x}{-} g_d \to g_d, g_d$

$r = 14$ is the reaction distance;
$\sigma(g_4, g_t) = xxx$ and $\sigma(g,h) = \emptyset$ for all other $(g,h) \in \gamma$.

The M system $\mathcal{M}_0$ passes a (possibly infinite) sequence of configurations described below. Since the system is nondeterministic, the provided description captures its most probable development, with possible statistical deviations:

1. In the initial configuration there is a single seed tile q_0 for cellular membrane. There are objects a in the environment in concentration 0.1 per cubic unit.
2. In the first step, the auxiliary rod s_0 is created and attached to the centre of the seed tile q_0 so that it is perpendicular to it. Simultaneously, five cellular membrane tiles q_2 are created by the rule $a^8 \to q_2$, and attached to five edge connectors of the tile q_0. Their other connectors at mutually matching positions with glues g_b attach together, too.
3. In the second step, tile q_3 is created and attached perpendicularly to rod s_0, hence it is parallel with q_0. Simultaneously, five more tiles q_2 are connected to the existing tiles q_2, forming subsequently the closed structure playing the role of cellular membrane.

4. In the third step, pentagonal tile q_1 concludes the cellular membrane building phase, completing the dodecahedron-shaped cell ("soccer ball") and enclosing inside rather large number of objects a (over a hundred of thousands). Simultaneously, an analogous process of membrane creation on a smaller scale starts from tile q_3 to which five tiles q_4 are attached, and the nuclear membrane formation is completed in next two steps.
5. During the previous two steps, the reaction proteins p_2 on tile q_2 already started to catalyze the reaction of objects a to b applying the rule $p_2a \to p_2b$. As each tile q_2 contains two proteins, and the cellular membrane contains 10 tiles q_2, 20 objects b is produced at each step within the cell. An analogous process runs in the nuclear membrane with proteins on tiles q_4 and objects c.
6. The objects b allow for creation of the microtubules s_1 and s_2, which can connect to point connectors in centres of "polar" tiles q_0 and q_1 of the cellular membrane. A non-deterministic growth of cytoskeleton towards the interior of the cell starts, consuming objects b as it continues.
7. When the microtubules s_1 and s_2 of the growing cytoskeleton reach the nuclear membrane, they can attach to its polar tiles q_3, each such contact releasing three objects x (as defined in the mapping σ).
8. Each object x can break one bond of a pair of tiles q_2 (or q_4) in the equatorial part of both membranes.
9. When all these bonds (10 in each membrane) are broken, the cell splits into two parts, each containing a half of the outer membrane, a part of the cytoskeleton growing from it, and the corresponding half of the nucleus.
10. The same growth rules as in the second and the third step now allow to complete both halves into two complete cells, pushing them apart as they grow. Before concluding this process, however, two important events take place: (i) both open halves of the cellular membrane are replenished with objects a from the environment, and (ii) open nucleus releases a large amount of accumulated objects c, while replenishing a's. Objects c diffuse into both halves of the cell, deactivating objects x via the rules $[p_1cx \to [p_1aa$, $[p_3cx \to [p_3aa$ and $cx[p_3 \to aa[p_3$.
11. After two steps during which the two new cells are completed, cytoskeleton growth can continue and the process of the "cell" division is repeated.

In such a way, the cells in the M system continue their division and the whole system is in a dynamical homeostasis: if environmental resources (represented by objects a) are unlimited, then so is the growth of the system. In the limited environment the growth would stop after exhaustion of the environmental resources.

Notice that, as the model is nondeterministic, the mitosis can be sometimes blocked, for instance because the objects x regulating the division process may start another nucleus division before the whole cell is ready to divide. This synchronization disorder in division control makes the cell dysfunctional and incapable of further mitosis.