A note on descriptional complexity of simple eco-grammar systems

Petr Sosík*

Institute of Computer Science, Silesian University,
Bezručovo náměstí 13, 746 01 Opava, Czech Republic,
E-mail:Petr.Sosik@fpf.slu.cz

Technical report No. 2000/2

Abstract

Descriptional complexity of eco-grammar systems and context-free grammars is compared, using standard complexity measures *Var*, *Prod* and *Symb*. It is shown that even for case of simple eco-grammar systems without internal representation of agents, the relation $<_4$ holds for *Var* and *Prod*, and the relation $<_3$ holds for *Symb*.

1 Basic definitions

In further text we will use the following basic notation (for elements of formal language theory see e.g. [5]): For an alphabet V, we denote by V^* the free monoid generated by V under the operation of concatenation; the empty string is denoted λ and $V^+ = V^* - \{\lambda\}$. For a word x over an aphabet V, $|x|$ denotes the length of x. For a set of context-free (CF) rules P we denote $\mathrm{dom}(P) = \{A \mid A \to \alpha \in P\}$.

A *0L scheme* is a construct $H = (V, P)$, where V is a finite alphabet and P is a complete set of context-free rules over V, that is $\mathrm{dom}(P) = V$. The rules are used in a parallel manner.

Definition 1.1 *An extended simple eco-grammar system of degree n, $n \geq 1$, is a construct $\Sigma = (V_E, P_E, R_1, \ldots, R_n, w, \Delta)$, where*

- *$E = (V_E, P_E)$ is a 0L scheme*
- *R_i is a finite set of context-free rewriting rules over V_E,*
- *$w \in V_E^*$*

*Research supported by the Grant Agency of Czech Republic, grants No. 201/98/P047 and 201/99/1086.

- $\Delta \subseteq V_E$.

In this construct V_E is the alphabet and P_E is the set of evolution rules of the environment. R_i represents the i-th agent, for $1 \leq i \leq n$, this is the set of its action rules. String w is the axiom — initial state of the system. The subset Δ is the terminal alphabet.

The systems changes its state by a simultaneous actions of all the agents and by a parallel rewriting of the remaining state symbols according to P_E.

Definition 1.2 *Let $\Sigma = (V_E, P_E, R_1, \ldots, R_n, w, \Delta)$ be an extended simple eco-grammar system, let $x \in V_E^+$, $y \in V_E^*$.*

We say that x directly derives y, written as $x \Longrightarrow_\Sigma y$, if

- $x = z_1 X_1 z_2 X_2 \ldots z_n X_k z_{n+1}$, $z_i \in V_E^*$, $1 \leq i \leq n+1$, $X_j \in V_E$, $1 \leq j \leq n$;
- $y = z_1' y_1 z_2' y_2 \ldots z_k' y_n z_{n+1}'$, $z_i' \in V_E^*$, $1 \leq i \leq n+1$, $y_j \in V_E^*$, $1 \leq j \leq n$;
- *there exists a permutation of agents $R_{j_1}, \ldots, R_{j_n}$ such that $X_i \to y_i \in R_{j_i}$, for $1 \leq i \leq n$,*
- *either $z_i = \lambda$ or $z_i \Longrightarrow_E z_i'$ for each i, $1 \leq i \leq n+1$, where $E = (V_E, P_E)$.*

Definition 1.3 *The language generated by an extended simple eco-grammar system $\Sigma = (V_E, P_E, R_1, \ldots, R_n, w, \Delta)$ is*

$$L(\Sigma) = \{v \in \Delta^* \mid w \Longrightarrow_\Sigma^* v\},$$

where $\Longrightarrow^*$ is the reflexive and transitive closure of $\Longrightarrow$.

Definition 1.4 *Let $G = (N, T, P, S)$ be an context-free grammar. Consider the following natural measures of descriptional complexity of Σ :*

$$\begin{aligned} Var(G) &= card(N), \\ Prod(G) &= card(P), \\ Symb(G) &= \sum_{A \to x \in P} (|x| + 2). \end{aligned}$$

Definition 1.5 *Let $\Sigma = (V_E, P_E, R_1, \ldots, R_n, w, \Delta)$ be an extended simple eco-grammar system. We define*

$$\begin{aligned} Var(\Sigma) &= card(V_E - \Delta), \\ Prod(\Sigma) &= card(P_E) + \sum_{i=1}^{n} card(R_i), \\ Symb(\Sigma) &= \sum_{A \to x \in P_E} (|x| + 2) + \sum_{i=1}^{n} \sum_{B \to y \in R_i} (|y| + 2). \end{aligned}$$

For a language L, a measure $M \in \{ Var, Prod, Symb\}$ and a class Y of grammatical constructs, we define the complexity of L as

$$M_Y(L) = \min\{M(\Gamma) \mid L(\Gamma) = L \text{ and } \Gamma \in Y\}.$$

For a family of languages $\mathcal{L}$, which can be derived by elements of different classes of grammatical constructs X and Y, we define the following relations:

$Y <_1 X\ (M)$	iff	there is a language $L \in \mathcal{L}$ such that $M_Y(L) < M_X(L)$,
$Y <_2 X\ (M)$	iff	for all $n \geq 1$, there is an $L \in \mathcal{L}$ such that $M_X(L_n) - M_Y(L_n) > n$,
$Y <_3 X\ (M)$	iff	there are $L_n \in \mathcal{L}$ for $n \geq 1$ such that $\lim_{n\to\infty} \frac{M_Y(L_n)}{M_X(L_n)} = 0$,
$Y <_4 X\ (M)$	iff	there are $p \geq 1$ and languages $L_n \in \mathcal{L}$, $n \geq 1$ such that $M_Y(L_n) \leq p$ and $M_X(L_n) > n$.

2 Main results

We denote CF the class of context-free grammars, SEG the class of simple extended eco-grammar systems. The following complexity relations hold with respect to the family of context-free languages.

Theorem 2.1 *SEG $<_4$ CF (Var)*

Proof. Consider the context-free language

$$L_n = \bigcup_{k=1}^{n} \{a^{ki} b^k a^i \mid i \geq 1\},$$

for which $Var_{\mathrm{CF}}(L_n) = n + 1$ ([5], vol. 2, p. 187). However, $Var_{\mathrm{SEG}}(L_n) = 4$, because $L_n = L(\Sigma)$ for

$$\Sigma = (\{A, R, S, T, a, b\}, P_E, R_1, \ldots, R_{n+1}, ST^n, \{a, b\}),$$

where

$$\begin{aligned}
P_E &= \{X \to X \mid X \in V_E\}, \\
R_i &= \{S \to R^{n-i} A^i b^i A, R \to R, A \to aA, A \to a, R \to \lambda, T \to \lambda\}, \quad 1 \leq i \leq n, \\
R_{n+1} &= \{R \to R, A \to aA, A \to a, R \to \lambda, T \to \lambda\}.
\end{aligned}$$

$\square$

Theorem 2.2 *SEG $<_4$ CF (Prod)*

Proof. Consider the context-free language

$$L_n = \{a^i b a^j d c^n \mid 0 \leq i + j \leq n\},$$

for which $Prod_{\mathrm{CF}}(L_n) \geq \log_2(n+1)$ (proof according to [1]). However, $Prod_{\mathrm{SEG}}(L_n) = 4$, because $L_n = L(\Sigma)$ for

$$\Sigma = (\{A, S, a, b, c, d\}, P_E, R_1, S, \{a, b, c, d\}),$$

where

$$\begin{aligned} P_E &= \{A \to a, A \to \lambda, S \to S, a \to a, b \to b, c \to c, d \to d\}, \\ R_1 &= \{S \to A^{n+1} d c^n, A \to b\}. \end{aligned}$$

□

Theorem 2.3 *SEG* $<_3$ *CF (Symb)*

Proof. Consider the context-free language

$$L_n = \{a_{i_r} a_{i_{r-1}} \dots a_{i_1} c_i b_{i_1} b_{i_2} \dots b_{i_r} \mid r \geq 1,\ i = i_s \text{ for some } s,\ 1 \leq s \leq r\}$$

over alphabet $\{a_1, \dots, a_n, b_1, \dots, b_n, c_1, \dots, c_n\}$, for which $Symb_{\mathrm{CF}}(L_n) > 3n^2 - 3n$ ([4]). However, $Symb_{\mathrm{SEG}}(L_n) \leq 41n + 9$, because $L_n = L(\Sigma)$ for

$$\Sigma = (\{C, D, S_i, T_i, a_i, b_i, c_i \mid 1 \leq i \leq n\}, P_E, R_1, \dots, R_{n+1}, BS_1 S_2 \dots S_n, \{a_i, b_i, c_i \mid 1 \leq i \leq n\}),$$

where

$$\begin{aligned} P_E &= \{X \to X \mid X \in V_E\}, \\ R_i &= \{C \to a_i C b_i, D \to c_i, S_i \to S_i, T_i \to T_i, S_i \to \lambda, T_i \to \lambda\}, \quad 1 \leq i \leq n, \\ R_{n+1} &= \{C \to D, S_i \to T_i, T_i \to T_i, T_i \to \lambda \mid 1 \leq i \leq n\}. \end{aligned}$$

We have

$$\sum_{A \to x \in P_E} (|x|+2) = 3(5n+2) \qquad \sum_{i=1}^{n} \sum_{B \to y \in R_i} (|y|+2) = 18n \qquad \sum_{B \to y \in R_{n+1}} (|y|+2) = 8n+3.$$

Therefore

$$\lim_{n \to \infty} \frac{Symb_{\mathrm{SEG}}(L_n)}{Symb_{\mathrm{CF}}(L_n)} \leq \lim_{n \to \infty} \frac{41n + 9}{3n^2 - 3n} = 0.$$

□

3 Discussion

Comparison of the descriptional complexity of eco-grammar systems and Lindenmayer systems would be probably more appropriate. Nevertheless we conjecture that the results would be the same as in the above case of context-free grammars.

References

[1] W. Bucher, K. Culik II, H.A. Maurer, D. Wotschke: Concise description of finite languages. *Theoretical Computer Science* **14** (1981), 227–246.

[2] J. Csima: Remarks on two variants of simple eco-grammar systems. *Acta Cybernetica,* to appear.

[3] E. Csuhaj-Varjú, J. Kelemen, A. Kelemenová, Gh. Păun: Eco-grammar systems – a grammatical framework for life-like interactions. *Artificial Life* **3** (1997), 1–28.

[4] J. Kelemen: Measuring cognitive resources use (a grammatical approach). *Computers and AI* **8** (1989), 29–42.

[5] A. Salomaa, G. Rozenberg (Eds.): *Handbook of Formal Languages.* 3 volumes. Springer-Verlag, Berlin, 1997.